AP Calculus AB

Extrema on an Interval

1) a) abs max @ $x=0$
abs min @ $x=-2, 2$

b) abs max @ $x=c$

c) abs max @ $x=b$
abs min @ $x=c_2$

2)

a) $x(t) = 3t^4 + 4t^3 - 6t^2$
$x'(t) = 12t^3 + 12t^2 - 12t = 0$
$12t(t^2 + t - 1) = 0$
$t=0$ $\quad t = \frac{-1 \pm \sqrt{1-4(1)(-1)}}{2}$
$t = \frac{-1 \pm \sqrt{5}}{2}$

b) $g(t) = 5t^{2/3} + t^{5/3}$
$g'(t) = \frac{10}{3}t^{-1/3} + \frac{5}{3}t^{2/3} = 0$
$t^{-1/3}\left(\frac{10}{3} + \frac{5}{3}t\right) = 0$
$\frac{1}{t^{1/3}} \rightarrow t=0$ $\quad \frac{10}{3} = -\frac{5}{3}t$
$t = -2$

c) $f(x) = x \ln x$
$f'(x) = x \cdot \frac{1}{x} + \ln x$
$= 1 + \ln x = 0$
$\ln x = -1$
$x = e^{-1}$

d) $g(\theta) = \frac{1}{2}\theta - \cos\theta \quad (0, 2\pi)$
$g'(\theta) = \frac{1}{2} + \sin\theta = 0$
$\sin\theta = -\frac{1}{2}$
$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

e) $f(x) = x\,e^{2x}$
$f'(x) = x(2e^{2x}) + e^{2x}$
$= e^{2x}(2x+1) = 0$
$x = -\frac{1}{2}$

3) a) $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$
$f'(x) = 6x^2 - 6x - 12 = 0$
$6(x^2 - x - 2) = 0$
$(x-2)(x+1) = 0$
$x = 2 \quad x = -1$

x	$f(x)$	
-2	$-16 - 12 + 24 + 1 = -3$	
-1	$-2 - 3 + 12 + 1 = 8$	← MAX
2	$16 - 12 - 24 + 1 = -27$	← MIN
3	$54 - 27 - 36 + 1 = -8$	

b) $f(x) = (x^2-1)^3$ on $[-1, 2]$
$f'(x) = 3(x^2-1)^2 \cdot 2x = 0$
$3(x^2-1)^2 = 0 \quad 2x = 0$
$x = 1, -1 \quad x = 0$

x	$f(x)$	
-1	0	
0	-1	← MIN
1	0	
2	27	← MAX

4) $f(x) = x^3 + ax^2 + b$ $\quad f'(x) = 3x^2 + 2ax$

$f(2) = 8 + 4a + b = 3$ $\quad f'(2) = 12 + 4a = 0$

$4a = -12$

$a = -3$

$4a + b + 8 = 3$

$-12 + b + 8 = 3$

$b = 7$

$f(x) = x^3 - 3x^2 + 7$

5) $y = x^2 - 4x + 3$ $\quad [0, 5]$

$y' = 2x - 4 = 0$

$2x = 4$

$x = 2$

x	y
0	3
2	-1
5	8

← MAX